

靜宜大學 104 學年度碩士班招生考試試題  
學系：財務與計算數學系 科目：線性代數

- (10 pts) Compute  $A(B + 3C) + (5B - 3A)C + CB - 5BC$  where  $A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ ,  
 $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ , and  $C = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$ .
- (10 pts) Find the matrix  $A$  if  $(A^T - 3I)^{-1} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ .
- (10 pts) Given vectors  $\mathbf{a}_1 = [1, 0, 1]^T$ ,  $\mathbf{a}_2 = [2, -1, 1]^T$ , and  $\mathbf{a}_3 = [0, 1, 1]^T$ , find a vector  $\mathbf{b}$ , that is not a linear combination of  $\mathbf{a}_1$ ,  $\mathbf{a}_2$ , and  $\mathbf{a}_3$ .
- (10 pts) Compute trace and determinant of the matrix  $A = \begin{bmatrix} 0 & 2 & -2 & 0 \\ 2 & 0 & 0 & 3 \\ 0 & 3 & 2 & 6 \\ 3 & 0 & 0 & 6 \end{bmatrix}$ .
- (10 pts) Compute the projection of the vector  $\mathbf{a} = [-2, 1, 1]^T$  on the vector  $\mathbf{b} = [1, 0, 1]^T$ .
- (10 pts) For the matrix  $A = \begin{bmatrix} 3 & 1 & 1 \\ -4 & -2 & -5 \\ 2 & 2 & 5 \end{bmatrix}$ , find the eigenvalues, eigenvectors, and an invertible matrix  $P$  such that  $P^{-1}AP$  is diagonal.
- (10 pts) If  $T: P_2 \rightarrow R^2$  is a linear transformation such that  $T(x + 1) = (2, -1)$  and  $T(x + x^2) = (1, 0)$ , then find  $T(2 + x - x^2)$ , where  $P_2$  is the vector space of polynomials with degree less than or equal to 2.
- (10 pts) Find the dimension of the subspace spanned by the vectors  $\mathbf{w} = [1, -1, 4, -5]^T$ ,  $\mathbf{x} = [2, 1, 5, -1]^T$ ,  $\mathbf{y} = [0, 1, -1, 3]^T$ , and  $\mathbf{z} = [3, 4, 5, 6]^T$ .
- (10 pts) If  $A$  and  $B$  are  $n \times n$  matrices,  $AB = -BA$ , and  $n$  is odd, show that either  $A$  or  $B$  has no inverse.
- (10 pts) Find a matrix  $A$  with right inverse but the system  $Ax = \mathbf{0}$  has infinitely many solutions.