1.

$$D = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}.$$

(a)(5points) Find the characteristic polynomial for D
(b)(5points) Find the eigenvalues for D
(c)(10points) Find D^2-5D and $D^{22}-5D^{21}+4D^{20}+D^4-5D^3+4D^2+2D$

2. (20points) Find $A+B$, $A-B$, AB and BA where the matrices

$$A = \begin{bmatrix} 7 & 1 & 3 \\ 2 & 10 & 4 \\ 3 & 3 & 9 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 4 & 1 \\ 2 & 1 & -6 \end{bmatrix}.$$

3. (10points) (a) Write down the definition of $tr(A)$, the trace of a $n \times n$ square matrix A.
(10points) (b) If A and B are both $n \times n$ real matrices, prove that $tr(AB) = tr(BA)$ where $tr(AB)$ represents the trace of the square matrix AB.

4. (20points) Let $S = \{u_1, u_2, u_3\}$ be the basis for R^3, where $u_1 = (1,2,2)$, $u_2 = (1,-8,2)$, and $u_3 = (2,4,1)$. Use the Gram-Schmidt process to transform S to an orthonormal basis for R^3.

5. Let $F : R^3 \to R^2$ and $G : R^2 \to R^3$ be linear transformations defined by

$$F\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 + 3x_3 \\ 2x_1 + x_2 + x_3 \end{bmatrix}$$

$$G\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 5x_2 \\ x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}.$$

(a)(10points) Give a formula for $G \circ F$.
(b)(10points) Find matrices A, B and C such that $F(x) = Ax$, $G(x) = Bx$, and $[G \circ F](x) = Cx$.